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Mass splittings within heavy baryon isospin multiplets in chiral perturbation theory

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Abstract: We calculate the mass splittings within the heavy baryon isospin multiplets $\Sigma_{c(b)}$ and $\Xi'_{c(b)}$ in chiral perturbation theory to leading one-loop order. The pattern of the mass splittings in the Σ_c iso-triplet, which is different from that of any other known isospin multiplet, can be explained. We predict $m_{\Xi_c'^+} - m_{\Xi_c'^0} = -0.2 \pm 0.6\,\mathrm{MeV}$, $m_{\Xi_b'^0} - m_{\Xi_b'^-} = -4.0 \pm 1.9\,\mathrm{MeV}$ and the mass of the Σ_b^0 to be 5810.3 ± 1.9 MeV.

Keywords: NLO Computations, Chiral Lagrangians, QCD.

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Contents

1. Introduction

Mass splittings within isospin multiplets of hadrons appear due to both the mass difference between the u and d quarks and electromagnetic (em) effects. Since the d quark is heavier than the u, usually the hadron with more d quarks is heavier within one isospin multiplet. For instance, the neutron (udd) is heavier than the proton (uud), and the $K^0(d\bar{s})$ is heavier than the $K^+(u\bar{s})$. There is only one exception to this pattern, the Σ_c iso-triplet, consisting of the $\Sigma_c^{++}(cuu)$, the $\Sigma_c^+(cud)$ and the $\Sigma_c^0(cdd)$. The mass splittings within the Σ_c iso-triplet are measured [1]

$$\begin{aligned}\Delta_{1c} &\equiv m_{\Sigma_c^+} - m_{\Sigma_c^0} = -0.9 \pm 0.4 \text{ MeV},\\ \Delta_{2c} &\equiv m_{\Sigma_c^{++}} - m_{\Sigma_c^0} = 0.27 \pm 0.11 \text{ MeV}\,.\end{aligned} \tag{1.1}$$

Remarkably, the state with two u quarks has the largest and the one with a u and a d quark has the smallest mass.

Only recently some of the bottom cousins of the Σ_c, the $\Sigma_b^{\pm}$, were observed by the CDF Collaboration [2]. Their masses are for the buu state $m_{\Sigma_b^+} = 5807.8 \pm 2.7\,\text{MeV}$ and for the bdd state $m_{\Sigma_b^-} = 5815.2 \pm 2.0\,\text{MeV}$, respectively — their neutral partner Σ_b^0 has not been observed yet. Thus, here the natural ordering of the states seems to be restored. On the other hand, heavy quark symmetry relates baryons containing a b quark to those with a c quark, which makes this different pattern even more puzzling. In this work we investigate the origin of these patterns, together with those in the $\Xi'_{c(b)}$ doublets, using chiral perturbation theory (CHPT) and heavy quark symmetry. In this way we can include both

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sources of isospin violation in a way consistent with QCD [3]. Our work is a straightforward extension of analogous studies for the nucleon [4, 5] (for pioneering studies, see [6, 7]).

The isospin splittings for heavy baryons were already studied in various quark models in refs. [8 – 16]. The advantages of our investigation are that (*i*), since we use an effective field theory, the theoretical uncertainty can be estimated, (*ii*) for the first time meson loop corrections are considered — they turn out to be numerically significant for the charm baryons, and (*iii*) this study investigates all isospin splittings at the same time. We compare our results to those of the quark models below.

Our paper is organized as follows. In section 2, we construct the SU(3) chiral effective Lagrangians responsible for the mass corrections of the sextet heavy baryons at order $\mathcal{O}(p^2)$. Here, p denotes the expansion parameter of the underlying effective field theory. The strong Lagrangian is proportional to the quark mass and the em Lagrangian is constructed including virtual photons. As we will show, the operator structure of the effective field theory for the heavy-light baryons is richer than the one of the light quark sector since heavy and light quarks must be treated differently. Calculations up to $\mathcal{O}(p^3)$ are performed in section 3. A brief summary is given in the last section. Some technicalities are relegated to the appendices.

2. The chiral effective Lagrangians

In CHPT the quark mass difference enters explicitly through the quark mass matrix. The inclusion of the virtual photons has been first considered systematically for the three-flavor case in ref. [17]. Chiral Lagrangians with virtual photons have been constructed for the study of isospin symmetry breaking phenomena in mesons and baryons with light up and down quarks (see, e.g., [4, 5, 18 – 29] for an incomplete list). Recently, this technique was used to study the interaction between Goldstone bosons and heavy-light mesons and the isospin breaking decay width of the $D_{s0}^*(2317)$ [30].

In this paper, we use the technique of SU(2) chiral perturbation theory to study the mass splittings within the heavy baryon isospin multiplets. The u, d quark masses and the electric charge e are counted as small quantities. They are booked as $m_u, m_d \sim \mathcal{O}(p^2)$, and $e \sim \mathcal{O}(p)$ as usual, where p denotes a small momentum with respect to the typical hadronic scale of about 1 GeV. Both types of isospin-violating effects $\sim (m_u - m_d)$ and $\sim e$ are taken into account here in a systematic manner to the order $\mathcal{O}(p^3)$. These effects can be accounted for in three-flavor CHPT to study the mass splittings within the SU(3) multiplets systematically. However, similar to the case of CHPT for light baryons, the SU(3) breaking contributions from the kaon-baryon and eta-baryon loops are large, see e.g. [31 – 33], which makes the convergence of the chiral expansion problematic. We will therefore treat each heavy baryon isospin multiplet separately to $\mathcal{O}(p^3)$ in two-flavor CHPT, and relate the low-energy constants (LECs) to $\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ through SU(3) relations (see also the discussion in section 3). On the other hand, the splittings within the multiplets are well behaved. This procedure is equivalent to starting from the SU(3) Lagrangian and to calculate only the leading order SU(2) loops, i.e., the pion-baryon loops.

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In this section, we will construct the SU(3) chiral effective Lagrangians pertinent to the mass corrections of the heavy baryons to order $\mathcal{O}(p^3)$ (for similar works considering the strange quark as heavy, see refs. [34 – 37]).

In order to construct the chiral effective Lagrangians, the following building blocks are necessary (we employ the standard nonlinear realization of chiral symmetry)

$$\begin{aligned} U &= \exp\left(\frac{\sqrt{2}i\phi}{F_\pi}\right), \quad u^2 = U, \\ u_\mu &= iu^\dagger \nabla_\mu U u^\dagger, \\ \nabla_\mu U &= \partial_\mu U - iQA_\mu U + iUQA_\mu, \\ \chi_+ &= u^\dagger \chi u^\dagger + u\chi u, \\ Q_\pm &= \frac{1}{2}\left(u^\dagger Q u \pm u Q u^\dagger\right), \end{aligned} \tag{2.1}$$

where F_π is the pion decay constant,[1] ϕ collects the Goldstone boson fields

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \tag{2.2}$$

and A_μ is the photon (em) field. The diagonal quark mass matrix and the charge matrix are

$$\begin{aligned} \chi &= 2B_0 \cdot \mathrm{diag}\left\{m_u, m_d, m_s\right\}, \\ Q &= e \cdot \mathrm{diag}\left\{2/3, -1/3, -1/3\right\}, \end{aligned} \tag{2.3}$$

in terms of $B_0 = |\langle 0|\bar{q}q|0\rangle|/F_\pi^2$ and the elementary electric charge e ($e > 0$). Under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, u_μ, χ_+ and $Q_\pm$ transform as

$$\mathcal{O} \to h\mathcal{O}h^\dagger, \tag{2.4}$$

where the compensator field h is an element of the conserved vector subgroup $\mathrm{SU}(3)_V$.

The Σ_c iso-triplet and the Ξ_c' iso-doublet belong to the symmetric sextet and the Λ_c^+ and the Ξ_c doublet belong to the anti-symmetric triplet in the flavor SU(3) classification. We use the following matrix representation in accordance with the notation of refs. [38, 39]

$$B_{6c} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\Sigma_c^{++} & \Sigma_c^+ & \Xi_c'^+ \\ \Sigma_c^+ & \sqrt{2}\Sigma_c^0 & \Xi_c'^0 \\ \Xi_c'^+ & \Xi_c'^0 & \sqrt{2}\Omega_c^0 \end{pmatrix}, \quad B_{\bar{3}c} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}. \tag{2.5}$$

Under $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, the transformation laws of the charmed baryon fields are [38]

$$B_{6c} \to hB_{6c}h^T, \quad B_{\bar{3}c} \to hB_{\bar{3}c}h^T, \tag{2.6}$$

[1] Strictly speaking, this should be the pion decay constant in the chiral limit. To the accuracy we are working, however, we do not need to differentiate this from its physical value.

where h^T is the transpose of h. The matrices for the bottom baryons can be obtained replacing c by b and decreasing the electric charge of every state by one unit.

Let $B_{\bar{3}Q}$, B_{6Q} $(Q = c, b)$ denote the heavy baryon fields, and $\mathring{m}_{\bar{3}Q}$, $\mathring{m}_{6Q}$ their masses in the chiral limit, respectively. Analogous to the effective chiral Lagrangian for the pion-nucleon system, the lowest order Lagrangian involving the sextet and anti-triplet heavy baryon fields can be written as [38, 40]

$$\begin{aligned}\mathcal{L}^{(1)} = & \frac{1}{2}\left\langle \bar{B}_{\bar{3}Q}(i\gamma_\mu \tilde{D}^\mu - \mathring{m}_{\bar{3}_Q})B_{\bar{3}Q}\right\rangle + \left\langle \bar{B}_{6Q}(i\gamma_\mu \tilde{D}^\mu - \mathring{m}_{6_Q})B_{6Q}\right\rangle \\ & + \frac{1}{2}g_1\left\langle \bar{B}_{6Q}\gamma_\mu\gamma_5\tilde{u}^\mu B_{6Q}\right\rangle + \frac{1}{2}g_2\left\langle \bar{B}_{6Q}\gamma_\mu\gamma_5\tilde{u}^\mu B_{\bar{3}Q}\right\rangle + h.c. + \frac{1}{2}g_3\left\langle \bar{B}_{\bar{3}Q}\gamma_\mu\gamma_5\tilde{u}^\mu B_{\bar{3}Q}\right\rangle . \quad (2.7)\end{aligned}$$

with

$$\tilde{D}_\mu = D_\mu - iQ_{B+}A_\mu, \quad \tilde{u}_\mu = u_\mu - 2Q_{B-}A_\mu \ . \quad (2.8)$$

The chiral covariant derivative on the baryon fields D_μ is given by

$$\begin{aligned} D_\mu &= \partial_\mu + \Gamma_\mu, \\ \Gamma_\mu &= \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right). \end{aligned} \quad (2.9)$$

The charge matrix of the heavy baryons Q_B, which gives the correct minimal coupling of the heavy baryons to photons, is constructed as

$$Q_B = 2Q + q_h\mathbb{I} = \begin{cases} e\cdot\mathrm{diag}\{2,0,0\}\,, & \text{for the charm baryons,} \\ e\cdot\mathrm{diag}\{1,-1,-1\}\,, & \text{for the bottom baryons,} \end{cases} \quad (2.10)$$

where q_h is the charge of the heavy quark, and $\mathbb{I}$ is a 3×3 unit matrix. $Q_{B\pm}$ is defined as

$$Q_{B\pm} = \frac{1}{2}\left(u^\dagger Q_B u \pm u Q_B u^\dagger\right).$$

At $\mathcal{O}(p^2)$ the strong Lagrangian pertinent to the corrections of the masses is given by the terms containing one power of the quark mass matrix

$$\mathcal{L}^{(2)}_{\text{str.}} = -\left\langle \bar{B}_Q\left(\alpha_1\chi_+ + \alpha_2\langle\chi_+\rangle\right)B_Q\right\rangle . \quad (2.11)$$

The em Lagrangian at $\mathcal{O}(p^2)$, parameterizing hard virtual photons, is more complicated than the one of the pion-nucleon system [4, 5]. The terms, which contribute to the mass corrections of the sextet heavy baryons and are quadratic in the light quark charge matrix read

$$\begin{aligned}\mathcal{L}^{(2)}_{\text{QQ}} = & -F_\pi^2\left\langle \bar{B}_{6Q}\left[\beta_0\left(Q_+^2 - Q_-^2\right) + \beta_1 Q_+\langle Q_+\rangle + \beta_2\left\langle Q_+^2 - Q_-^2\right\rangle + \beta_3\left\langle Q_+^2 + Q_-^2\right\rangle\right]B_{6Q}\right\rangle \\ & - F_\pi^2\beta_4\left\langle Q_+^T\bar{B}_{6Q}Q_+B_{6Q}\right\rangle . \end{aligned} \quad (2.12)$$

The em Lagrangian given above only deals with the hard photons exchanged between the light quarks in the heavy baryons. In addition we need to add terms that parameterize the em interactions between the heavy and a light quark. Since the heavy quark can be viewed as static, its charge q_h acts as a static background field that transforms as a scalar under

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$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$. It is easy to see that in this way we get one additional independent structure that contributes to the isospin splittings, namely

$$\mathcal{L}_{\rm em}^{(2)} = \mathcal{L}_{\rm QQ}^{(2)} - F_\pi^2 \beta_{1h} \left\langle \bar{B}_{6Q} Q_+ \langle q_h \mathbb{I} \rangle B_{6Q} \right\rangle . \tag{2.13}$$

As we will show below, it is this term which makes the mass splitting pattern within the Σ_c iso-triplet different from that within the Σ_b iso-triplet. The analogous mechanism is also present in the mentioned quark model calculations.

3. Mass splittings within the heavy baryon isospin multiplets

As mentioned above, our strategy is to relate the LECs for different heavy baryon isospin multiplets using SU(3) relations, but, since we are only after isospin splittings, calculate only the pion-baryon loop corrections. If we start from the SU(2) Lagrangians for each of the isospin multiplets, the chiral limit masses of the Σ_Q and Ξ'_Q are different, and the difference contributes to the SU(3) mass splittings of $\mathcal{O}(p^3)$. It is important to stress that the differences of the LECs between their SU(2) and SU(3) values starts contributing at order $\mathcal{O}(p^4)$ and thus is beyond the accuracy of our calculation.[2] Therefore, what we do is equivalent to use the SU(3) Lagrangians given in section 2 to do SU(2) calculations. Indeed, the number of independent operators matches for the two cases, since the contribution of the β_1 term to the baryon masses vanishes for SU(3) because the three-flavor quark charge matrix is traceless, and the β_0 term for SU(3) can be translated into the β_1 term for SU(2) by using the Cayley-Hamilton relation for 2×2 matrices

$$Q_+^2 - Q_-^2 = Q_+ \langle Q_+ \rangle + \frac{1}{2} \left(\langle Q_+^2 - Q_-^2 \rangle - \langle Q_+ \rangle^2 \right) . \tag{3.1}$$

3.1 Mass splittings to $\mathcal{O}(p^2)$

At leading order, there is no mass splitting within isospin multiplets. However, at next-to-leading order the terms in the $\mathcal{O}(p^2)$ Lagrangians generate mass splittings. From eqs. (2.11), (2.12), we get

$$\begin{aligned}
\Delta_{1c}^{(2)} &\equiv \left(m_{\Sigma_c^+} - m_{\Sigma_c^0} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 (\beta_0 - 2\beta_4 + 6\beta_{1h}), \\
\Delta_{2c}^{(2)} &\equiv \left(m_{\Sigma_c^{++}} - m_{\Sigma_c^0} \right)^{(2)} = 4\alpha_1 B_0 (m_u - m_d) + \frac{1}{3} F_\pi^2 e^2 (\beta_0 + \beta_4 + 6\beta_{1h}), \\
\Delta_{3c}^{(2)} &\equiv \left(m_{\Xi_c'^+} - m_{\Xi_c'^0} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 (\beta_0 - 2\beta_4 + 6\beta_{1h}) = \Delta_{1c}^{(2)},
\end{aligned} \tag{3.2}$$

for the charm baryons Σ_c and Ξ'_c. Similarly, for the bottom baryons we have

$$\begin{aligned}
\Delta_{1b}^{(2)} &\equiv \left(m_{\Sigma_b^0} - m_{\Sigma_b^-} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 (\beta_0 - 2\beta_4 - 3\beta_{1h}), \\
\Delta_{2b}^{(2)} &\equiv \left(m_{\Sigma_b^+} - m_{\Sigma_b^-} \right)^{(2)} = 4\alpha_1 B_0 (m_u - m_d) + \frac{1}{3} F_\pi^2 e^2 (\beta_0 + \beta_4 - 3\beta_{1h}), \\
\Delta_{3b}^{(2)} &\equiv \left(m_{\Xi_b'^0} - m_{\Xi_b'^-} \right)^{(2)} = 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} F_\pi^2 e^2 (\beta_0 - 2\beta_4 - 3\beta_{1h}) = \Delta_{1b}^{(2)}.
\end{aligned} \tag{3.3}$$

[2]The precise matching between two– and three-flavor versions of CHPT is discussed in refs. [41 – 43].

Baryons	Loops
Σ_c^{++}	$\Sigma_c^{++}\pi^0$, $\Sigma_c^{+}\pi^+$, $\Lambda_c^{+}\pi^+$
Σ_c^{+}	$\Sigma_c^{++}\pi^-$, $\Sigma_c^{0}\pi^+$, $\Lambda_c^{+}\pi^0$
Σ_c^{0}	$\Sigma_c^{+}\pi^-$, $\Sigma_c^{0}\pi^0$, $\Lambda_c^{+}\pi^-$
$\Xi_c^{\prime+}$	$\Xi_c^{\prime+}\pi^0$, $\Xi_c^{\prime0}\pi^+$, $\Xi_c^{+}\pi^0$, $\Xi_c^{0}\pi^+$
$\Xi_c^{\prime0}$	$\Xi_c^{\prime+}\pi^-$, $\Xi_c^{\prime0}\pi^0$, $\Xi_c^{+}\pi^-$, $\Xi_c^{0}\pi^0$

Table 1: Pion-baryon loops contributing to the charm baryon mass corrections.

Note that the α_1 and β_0 terms always appear in the same linear combination, which means that the strong contribution, the α_1 term, cannot be disentangled from the em contribution without additional information. This is completely analogous to the case of the neutron-proton mass splitting, see e.g. ref. [4]. The β_{1h} term has a different sign for charm baryons and bottom baryons, hence it is expected to induce a different interference pattern of the various contributions.

From the relations given above we find

$$\begin{aligned}
\left(m_{\Xi_c^{\prime+}} - m_{\Xi_c^{\prime0}}\right) - \left(m_{\Sigma_c^{+}} - m_{\Sigma_c^{0}}\right) &= \mathcal{O}(p^3), \\
\left(m_{\Xi_b^{\prime0}} - m_{\Xi_b^{\prime-}}\right) - \left(m_{\Sigma_b^{0}} - m_{\Sigma_b^{-}}\right) &= \mathcal{O}(p^3), \\
\left(m_{\Sigma_b^{+}} + m_{\Sigma_b^{-}} - 2m_{\Sigma_b^{0}}\right) - \left(m_{\Sigma_c^{++}} + m_{\Sigma_c^{0}} - 2m_{\Sigma_c^{+}}\right) &= \mathcal{O}(p^3).
\end{aligned} \tag{3.4}$$

The last relation is obtained invoking heavy quark symmetry.

3.2 Mass splittings to $\mathcal{O}(p^3)$

The first non-vanishing loop corrections to the baryon masses appear at order $\mathcal{O}(p^3)$. At this order formally both photon loops as well as pion-baryon loops contribute. Remarkably, QCD does not allow for a counterterm at this order and consequently the $\mathcal{O}(p^3)$ pieces of the these loops are finite. We start with the latter kind of loops that are to be constructed from two vertices of $\mathcal{O}(p)$. A complete list of loops is given in table 1 for the charm baryons considered here. Since in the power counting the pion mass difference is of the same order as the pion mass itself, in the loops we are to use the physical pion masses (for a detailed discussion of this point, see e.g. ref. [26]). On the other hand, to the order we are working, the masses to be used for the baryons of the same isospin multiplet are the same. Therefore, the leading SU(2) loop contributions to the mass corrections of the $\Xi_c^{\prime+}$ and $\Xi_c^{\prime0}$ as well as Σ_c^{++} and Σ_c^{0} are equal, as can be seen from table 1. Hence there is no loop correction for the corresponding mass differences at $\mathcal{O}(p^3)$. The pion-baryon loops for the Σ_c self-energies are shown in figure 1. Contrary to the case of the nucleon mass differences, here it is not straightforward to use the heavy baryon formalism to calculate the pion-baryon loops, since the pion–Λ_c contribution generates a cut. It is thus more convenient for us to evaluate the integrals using the covariant method of infrared regularization as derived by Becher and Leutwyler [44]. Some remarks on the method and the relevant integrals are given in appendix A.

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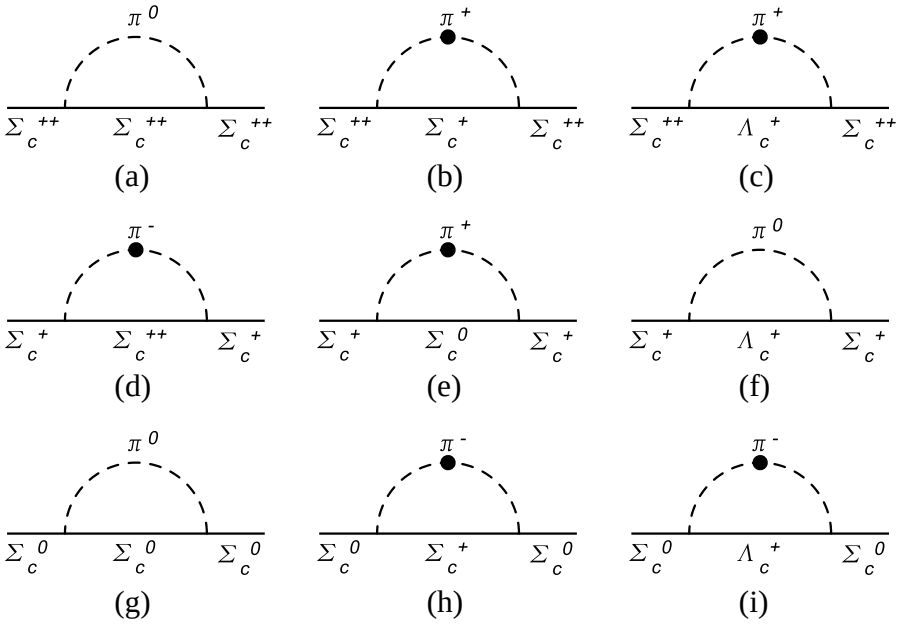

Figure 1: The pion-baryon loops contributing to the self-energies of the Σ_c^{++} (a, b, c), Σ_c^{+} (d, e, f) and Σ_c^{0} (g, h, i). The black dots in the charged pion propagators denote the electromagnetic insertions at $\mathcal{O}(p^2)$.

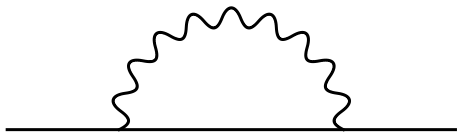

Figure 2: The photon-baryon loop.

The photon-baryon loops are shown in figure 2. Formally they also contribute at $\mathcal{O}(p^3)$. However, it can be shown that they vanish to this order. Since the baryon mass in the loop is equal to the mass of the external legs, for this loop we may use the integral representation of the heavy baryon formalism. Then the vanishing of the loop follows from the absence of a mass scale in the integral. This result also holds, when the infrared regularization is employed, as outlined in appendix A.

Therefore the mass splittings for the charm baryons Σ_c and Ξ_c' are to third order in the chiral expansion

$$m_{\Sigma_c^+} - m_{\Sigma_c^0} \equiv \Delta_{1c}^{(2)} + \Delta_{1c}^{\text{loop}}(m_{\Sigma_c}, m_{\Lambda_c}) + \mathcal{O}(p^4)\,, \tag{3.5}$$

$$m_{\Sigma_c^{++}} - m_{\Sigma_c^0} \equiv \Delta_{2c}^{(2)} + \mathcal{O}(p^4)\,, \tag{3.6}$$

$$m_{\Xi_c'^+} - m_{\Xi_c'^0} \equiv \Delta_{1c}^{(2)} + \mathcal{O}(p^4)\,. \tag{3.7}$$

The mass splittings for the bottom baryons Σ_b and Ξ_b' are

$$m_{\Sigma_b^0} - m_{\Sigma_b^-} \equiv \Delta_{1b}^{(2)} + \Delta_{1b}^{\rm loop}(m_{\Sigma_b}, m_{\Lambda_b}) + \mathcal{O}(p^4)\,, \tag{3.8}$$

$$m_{\Sigma_b^+} - m_{\Sigma_b^-} \equiv \Delta_{2b}^{(2)} + \mathcal{O}(p^4)\,, \tag{3.9}$$

$$m_{\Xi_b'^0} - m_{\Xi_b'^-} \equiv \Delta_{1b}^{(2)} + \mathcal{O}(p^4)\,, \tag{3.10}$$

where the results to $\mathcal{O}(p^2)$, the $\Delta^{(2)}$ were given in the previous subsection. As derived in appendix B, the loop functions to $\mathcal{O}(p^3)$ are given by

$$\begin{aligned}\Delta_{1Q}^{\rm loop}(m_{\Sigma_Q}, m_{\Lambda_Q}) = &-\frac{g_1^2}{32\pi F_\pi^2}\left(M_\pm^3 - M_0^3\right)\\ &+{\rm Re}\Sigma^{(c)}(m_{\Sigma_Q}; M_0, m_{\Lambda_Q}) - {\rm Re}\Sigma^{(c)}(m_{\Sigma_Q}; M_\pm, m_{\Lambda_Q})\,,\end{aligned} \tag{3.11}$$

with $M_0\,(M_\pm)$ the neutral (charged) pion mass. The explicit expressions for the loop functions are given in appendix B.

3.3 Numerical results

When calculating the loops, we take the physical values for the masses

$$\begin{aligned} M_\pm &= 139.57 \text{ MeV}, & M_0 &= 134.98 \text{ MeV},\\ m_{\Sigma_c} &= 2453.56 \text{ MeV}, & m_{\Lambda_c} &= 2286.46 \text{ MeV},\\ m_{\Sigma_b} &= 5811.5 \text{ MeV}, & m_{\Lambda_b} &= 5620.2 \text{ MeV}. \end{aligned} \tag{3.12}$$

The values of g_1 and g_2 can be estimated based on SU(6) [38]

$$g_1 = \frac{4}{3} g_A^{\rm ud}, \quad g_2 = -\sqrt{\frac{2}{3}} g_A^{\rm ud}, \tag{3.13}$$

where $g_A^{\rm ud}$ is the coupling constant for the single quark transition $u \to d$. A value of $g_A^{\rm ud} = 0.76$ gives the correct nucleon axial coupling constant $g_A = (5/3) g_A^{\rm ud} = 1.27$, correspondingly $g_1 = 1.02$. It also gives $g_2 = -0.62$, the absolute value of which is close to the empirical one $|g_2| = 0.58 \pm 0.04$ obtained from the measured decay width $\Gamma(\Sigma_c^{++} \to \Lambda_c^+ \pi^+) = 2.23 \pm 0.30\,{\rm MeV}$. The only unknown parameters are the LECs in the $\mathcal{O}(p^2)$ Lagrangians. There are effectively three

$$\begin{aligned} \tilde\gamma &\equiv 2\alpha_1 B_0 (m_u - m_d) + \frac{1}{6} e^2 F_\pi^2 \beta_0,\\ \tilde\beta_4 &\equiv e^2 F_\pi^2 \beta_4,\\ \tilde\beta_{1h} &\equiv e^2 F_\pi^2 \beta_{1h}. \end{aligned} \tag{3.14}$$

Totally there are four known isospin mass splittings of the sextet heavy baryons,[3]

$$\begin{aligned} m_{\Sigma_c^+} - m_{\Sigma_c^0} &= -0.9 \pm 0.4 \text{ MeV},\\ m_{\Sigma_c^{++}} - m_{\Sigma_c^0} &= 0.27 \pm 0.11 \text{ MeV}, \end{aligned}$$

[3]The first two mass splittings are given in PDG [1]. The last two mass splittings are evaluated by taking the difference of the masses given in PDG: $m_{\Sigma_b^+} = 5807.8 \pm 2.7\,{\rm MeV}$, $m_{\Sigma_b^-} = 5815.2 \pm 2.0\,{\rm MeV}$, $m_{\Xi_c'^+} = 2575.7 \pm 3.1\,{\rm MeV}$, and $m_{\Xi_c'^0} = 2578.0 \pm 2.9\,{\rm MeV}$.

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\begin{aligned}
m_{\Sigma_b^+} - m_{\Sigma_b^-} &= -7.4 \pm 3.4 \text{ MeV}, \\
m_{\Xi_c'^+} - m_{\Xi_c'^0} &= -2.3 \pm 4.2 \text{ MeV}.
\end{aligned} \tag{3.15}
$$

The first three will be taken to determine the LECs $\tilde{\gamma}$, $\tilde{\beta}_4$ and $\tilde{\beta}_{1h}$ because they have the smallest uncertainty.

Using the physical value of the pion decay constant $F_\pi = 92.4\,\text{MeV}$, we get the contribution of the loops to $m_{\Sigma_c^+} - m_{\Sigma_c^0}$

$$
\begin{aligned}
\Delta_{1c}^{\text{loop}} &= (-0.32 \pm 0.15) + (-0.41 \pm 0.06) \text{ MeV} \\
&= -0.73 \pm 0.16 \text{ MeV},
\end{aligned} \tag{3.16}
$$

where in the first line the numbers in the first parenthesis are from the π–Σ_c loops, and those in the second parenthesis from the π–Λ_c loops. The uncertainty of the loops is in general controlled by the expansion parameter of CHPT, namely $\xi = M/\Lambda_\chi$, with the chiral symmetry breaking scale $\Lambda_\chi \simeq 1\,\text{GeV}$. Since we do not really know the value of g_1, which enters in the π–Σ_c loops, for those we estimate the uncertainty conservatively as being of the order 4ξ. Since the coupling constant of the Σ_c to Λ_c and π can be extracted from experiment, for the second contribution we use directly the uncertainty that results from that extraction (see above). Note that the given uncertainty is at the same time numerically of order ξ. Thus the uncertainty estimate is consistent with what is expected from the chiral expansion. The LECs are then determined as

$$
\begin{aligned}
\tilde{\gamma} &= -2.5 \pm 1.1 \text{ MeV}, \\
\tilde{\beta}_4 &= 0.6 \pm 0.9 \text{ MeV}, \\
\tilde{\beta}_{1h} &= 2.6 \pm 1.1 \text{ MeV}.
\end{aligned} \tag{3.17}
$$

The two different contributions in eq. (3.16) are comparable, and they are considerably smaller than the individual strong and em contributions at $\mathcal{O}(p^2)$, see eqs. (3.2) and (3.17), showing good convergence of the chiral expansion. The mass splitting within the Ξ_c' doublet can be predicted

$$
m_{\Xi_c'^+} - m_{\Xi_c'^0} = m_{\Sigma_c^+} - m_{\Sigma_c^0} - \Delta_{1c}^{\text{loop}} = -0.2 \pm 0.4(\text{exp}) \pm 0.4(\text{th}) \text{ MeV}, \tag{3.18}
$$

where the first uncertainty coming from the uncertainty of $m_{\Sigma_c^+} - m_{\Sigma_c^0}$ is experimental, and the second one is theoretical. It comes from neglecting the $\mathcal{O}(p^4)$ contribution and was estimated by taking one half of the leading loop contribution.

Because the width of the Σ_b has not been measured so far, g_2 for the bottom baryons cannot be determined from the data. Taking the same values as before for g_1 and g_2, as dictated by heavy quark symmetry, we get the mass difference between Σ_b^0 and Σ_b^- at $\mathcal{O}(p^3)$

$$
\begin{aligned}
\Delta_{1b}^{\text{loop}} &= (-0.3 \pm 0.2) + (-0.6 \pm 0.1) \text{ MeV} \\
&= -0.9 \pm 0.2 \text{ MeV}.
\end{aligned} \tag{3.19}
$$

where in the first line the numbers in the first and the second parentheses are from the π–Σ_b loops and the π–Λ_b loops, respectively. The uncertainties were estimated as in case

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	$\Sigma_c^+ - \Sigma_c^0$	$\Sigma_c^{++} - \Sigma_c^0$	$\Xi_c'^+ - \Xi_c'^0$	$\Sigma_b^0 - \Sigma_b^-$	$\Sigma_b^+ - \Sigma_b^-$	$\Xi_b'^0 - \Xi_b'^-$
Exp. [1]	-0.9 ± 0.4	0.27 ± 0.11	-2.3 ± 4.2		-7.4 ± 3.4	
Our work	$-0.9 \pm 0.4^*$	$0.27 \pm 0.11^*$	-0.2 ± 0.6	-4.9 ± 1.9	$-7.4 \pm 3.4^*$	-4.0 ± 1.9
[8]	$-0.9 \pm 0.4^*$	$0.27 \pm 0.11^*$		-6.9 ± 1.1	$-7.4 \pm 2.3^*$	
[9]	-0.83	-0.01	-1.48			
[11]	-0.73	0.28	-3.20	-3.95	-6.12	-6.16
[10]	-0.2	1.4		-3.7	-5.6	
[12]	-0.5	3.0	-1.0	-5.6	-7.1	
[13]	-0.7	0.5	-1.2			
[14]	-0.40	0.84				
[15]	-0.36	1.20	-0.30	-2.51	-3.57	
[16]	-0.33	0.37	-0.20			

Table 2: Comparison of our results with the experimental data and the results from quark models (units are MeV). The numbers marked by $*$ are used as inputs.

of the charm baryons. According to eq. (3.3), the mass of the Σ_b^0, which has not been measured yet, is predicted to be

$$\begin{aligned} m_{\Sigma_b^0} &= \frac{1}{2}\left(m_{\Sigma_b^+} + m_{\Sigma_b^-} - \tilde{\beta}_4\right) + \Delta_{1b}^{\text{loop}} \\ &= 5810.3 \pm 1.8(\text{exp}) \pm 0.5(\text{th}) \text{ MeV}. \end{aligned} \tag{3.20}$$

We can also predict the mass difference between the $\Xi_b'^0$ and $\Xi_b'^-$

$$\begin{aligned} m_{\Xi_b'^0} - m_{\Xi_b'^-} &= \frac{1}{2}\left(m_{\Sigma_b^+} - m_{\Sigma_b^-} - \tilde{\beta}_4\right) \\ &= -4.0 \pm 1.8(\text{exp}) \pm 0.5(\text{th}) \text{ MeV}. \end{aligned} \tag{3.21}$$

In table 2, the results of our work are summarized and a comparison with those obtained in quark models is given.

4. Discussions and summary

In this work, we have calculated the mass splittings within the heavy baryon isospin multiplets $\Sigma_{c(b)}$ and $\Xi'_{c(b)}$ to $\mathcal{O}(p^3)$ in the chiral expansion. Our main results are given in eqs. (3.5–3.10) and in table 2. To arrive at these results, we constructed both the strong and the em Lagrangians at $\mathcal{O}(p^2)$ which are responsible for the mass corrections. In contrast to mass splittings in light quark baryon multiplets, there is an additional operator that describes the hard virtual photons exchanged between the heavy quark and light quarks accompanied by a LEC β_{1h}. Remarkably, this term has a different sign for the charm baryons and the bottom baryons. This is due to the fact that the sign of the electric

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charge of the charm quark is different from that of the bottom quark. It is the different interference between this term and the other terms that drives the mass splittings within the Σ_c iso-triplet to have a different pattern compared to any other known isospin multiplet. This leads one to expect that the isospin mass splittings in the charm hadrons are always different from those in the bottom hadrons even if the heavy quark symmetry were exact. Besides the heavy baryons considered in this paper, the D and B-meson mass splittings, $m_{D^\pm} - m_{D^0} = 4.78 \pm 0.10\,\mathrm{MeV}$ and $m_{B^0} - m_{B^\pm} = 0.37 \pm 0.24\,\mathrm{MeV}$ [1] are a nice example for the effect, although the ordering does not get changed here.

There is no loop contribution to the mass splitting between the two Ξ_c' baryons, and we predict $m_{\Xi_c'^+} - m_{\Xi_c'^0} = -0.2 \pm 0.6\,\mathrm{MeV}$. The present data for the masses of the Ξ_c' baryons are not accurate enough yet to test this prediction. For the Σ_b states, the β_{1h} term interferes constructively with the other terms and hence the loop corrections are less important. The mass of the Σ_b^0 and the mass difference $m_{\Xi_b'^0} - m_{\Xi_b'^-}$ are predicted to be 5810.3 ± 1.9 MeV and $-4.0 \pm 1.9\,\mathrm{MeV}$, respectively, which can be tested in future experiments.

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A. Infrared regularization

The finite masses of the baryons in the chiral limit spoil the correspondence between the loop expansion and chiral expansion if one uses conventional dimensional regularization [45]. Here we follow the infrared regularization (IR) method developed by Becher and Leutwyler [44] to overcome this problem. The IR method has been extended to two loops [46] (see also ref. [47]), and to the cases with spin-3/2 fields [48] and spin-1 fields [49, 50] (for reviews, see [51, 52]).

In the IR method, the scalar loop integral

$$H = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(M^2-k^2)\,[m^2-(P-k)^2]}. \tag{A.1}$$

with M and m being the masses of the pion and baryon, respectively, is split into two parts, one being infrared singular and the other being infrared regular, $H = I + R$. Only the singular part I, which is of order $\mathcal{O}(p)$, makes the expansion in loops to coincide with the chiral expansion, hence leads to a consistent power counting. The regular part R can be expanded in polynomials in M, hence it can be absorbed into the LECs order by order.

In any regularization such as the IR which has a consistent power counting for loops, the photon-baryon loops with each of the vertices being of the $\mathcal{O}(e)$ order should be counted as $\mathcal{O}(e^2p) = \mathcal{O}(p^3)$. In the photon-baryon loops, when taking $P^2 = m^2$, which is necessary for calculating chiral corrections to the baryon mass, there is no quantity of the order

of $\mathcal{O}(p)$ since the photon is massless. Therefore, the photon-baryon loops should vanish for calculating the mass shifts of baryons. Such an argument is supported by explicit calculations in the IR method.

The self-energy from any of the pion-baryon diagrams has the form

$$-i\Sigma^{(\mathrm{n})}_{\Sigma_Q}(\not{P};M,m) = -\frac{g_i^2}{4F_\pi^2}\int\frac{d^dk}{(2\pi)^d}\;\not{k}\gamma_5\frac{i}{(k^2-M^2+i\varepsilon)}\frac{i(\not{P}-\not{k}+m)}{[(P-k)^2-m^2+i\varepsilon]}\;\not{k}\gamma_5, \quad \text{(A.2)}$$

where (n) is a diagram label, P is the external momentum, and g_i $(i=1,2)$ are the $\Sigma_Q\Sigma_Q\pi$ and $\Sigma_Q\Lambda_Q\pi$ $(Q=c,b)$ coupling constants of the lowest order Lagrangian, eq. (2.7). After a few manipulations one gets

$$\Sigma^{(\mathrm{n})}_{\Sigma_Q}(\not{P};M,m) = \frac{g_i^2}{4F_\pi^2}(\not{P}+m)\left[M^2I(P^2)+(m-\not{P})\;\not{P}I^{(1)}(P^2)-\Delta_{\Sigma_Q}\right]. \quad \text{(A.3)}$$

In the IR method, the single baryon loop

$$\Delta_{\Sigma_Q} = i\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}$$

vanishes. The expressions for the loop functions $I(P^2)$ and $I^{(1)}(P^2)$ are given in appendix B.

Now let us focus on the Σ_c^{++}. Taking $\not{P}=m_{\Sigma_c^{++}}$, the expressions for the diagrams figure 1(a) and (b) are very simple because the term proportional to $(m-\not{P})$ does not contribute,

$$\Sigma^{(\mathrm{a})}(m_{\Sigma_c};M_0,m_{\Sigma_c}) = -\frac{g_1^2M_0^3}{32\pi F_\pi^2}, \quad \text{(A.4)}$$

where the chiral limit mass $\mathring{m}_{\Sigma_c}$ in the loop has been replaced by the physical mass since the contribution of the difference is of higher order. Note that up to $\mathcal{O}(p^3)$, we do not need to distinguish the masses of baryons with different electric charges in loops, so that m_{Σ_c}, instead of $m_{\Sigma_c^{++}}$, is used for the arguments of the loop function. Replacing the neutral pion mass M_0 by the charged pion mass $M_\pm$, the expression for diagram (b) is obtained. The contributions of the diagrams (a) and (b) do not depend on the baryon mass.

The expression for diagram (c) is much more complicated since the term proportional to $(m-\not{P})$ has a finite contribution

$$\Sigma^{(\mathrm{c})}(m_{\Sigma_c};M_\pm,m_{\Lambda_c}) = \frac{g_2^2}{4F_\pi^2}(m_{\Sigma_c}+m_{\Lambda_c})\left[M_\pm^2\bar{I}(m_{\Sigma_c}^2)+(m_{\Lambda_c}-m_{\Sigma_c})m_{\Sigma_c}\bar{I}^{(1)}(m_{\Sigma_c}^2)\right], \quad \text{(A.5)}$$

where the loop functions $I(P^2), I^{(1)}(P^2)$ are replaced by their finite parts $\bar{I}(P^2), \bar{I}^{(1)}(P^2)$ (subtracting the $\bar{\lambda}$ parts), and the divergences can be absorbed in the counterterms at $\mathcal{O}(p^4)$ which are not considered here. The expressions of the infrared singular parts of the loop integrals are given in appendix B. Here we use the expansion of the $\bar{I}(P^2)$ up to $\mathcal{O}(p)$. The chiral expansion of $\bar{I}(P^2)$ up to $\mathcal{O}(p)$, accounting for the cut due to the opening of the $\Lambda_c\pi$ channel, is

$$\bar{I}(P^2) = -\frac{\alpha}{16\pi^2}\left\{\Omega(2\ln\alpha-1)+\sqrt{\Omega^2-1}\left[\ln\left(\frac{\Omega+\sqrt{\Omega^2-1}}{\Omega-\sqrt{\Omega^2-1}}\right)-2i\pi\right]\right\}+O(\alpha^2), \quad \text{(A.6)}$$

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where

$$\alpha = \frac{M_\pm}{m_{\Lambda_c}}, \quad \Omega = \frac{P^2 - m_{\Lambda_c}^2 - M_\pm^2}{2M_\pm m_{\Lambda_c}}.$$

For $P^2 - m_{\Lambda_c}^2 \sim \mathcal{O}(p)$, which is the case for taking $P^2 = m_{\Sigma_c}^2$ since $m_{\Sigma_c} - m_{\Lambda_c} \simeq 170\,\mathrm{MeV}$, $I^{(1)}(P^2)$ starts from $\mathcal{O}(p^2)$, see eq. (B.6). The physical mass of the Σ_c is above the $\Lambda_c\pi$ threshold, correspondingly $\Omega > 1$.

Summing up the diagrams (a), (b) and (c), one gets the corrections to the mass of the Σ_c^{++} at $\mathcal{O}(p^3)$

$$\Delta m_{\Sigma_c^{++}}^{\mathrm{loop}} = -\frac{g_1^2(M_0^3 + M_\pm^3)}{32\pi F_\pi^2} + \mathrm{Re}\Sigma^{(\mathrm{c})}(m_{\Sigma_c}; M_\pm, m_{\Lambda_c}), \tag{A.7}$$

where Re represents taking the real part. Similarly for the Σ_c^+ and Σ_c^0, we have

$$\begin{aligned} \Delta m_{\Sigma_c^+}^{\mathrm{loop}} &= -\frac{2g_1^2 M_\pm^3}{32\pi F_\pi^2} + \mathrm{Re}\Sigma^{(\mathrm{c})}(m_{\Sigma_c}; M_0, m_{\Lambda_c}), \\ \Delta m_{\Sigma_c^0}^{\mathrm{loop}} &= \Delta m_{\Sigma_c^{++}}^{\mathrm{loop}}. \end{aligned} \tag{A.8}$$

B. Loop integrals

Defining

$$\begin{aligned} \alpha &= \frac{M}{m}, \quad s = P^2, \quad \Omega = \frac{s - m^2 - M^2}{2Mm}, \\ \bar{\lambda} &= \frac{m^{d-4}}{(4\pi)^2}\left[\frac{1}{d-4} - \frac{1}{2}\left(\ln(4\pi) + \Gamma'(1) + 1\right)\right], \end{aligned}$$

the infrared singular part of the loop integral

$$H(s) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{(M^2 - k^2)\left[m^2 - (P-k)^2\right]}, \tag{B.1}$$

is (the expression for $-1 < \Omega < 1$ is given in ref. [44])

$$\begin{aligned} I(s) &= \bar{I}(s) - \frac{s - m^2 + M^2}{s}\bar{\lambda}, \\ \bar{I}(s) &= -\frac{1}{16\pi^2}\frac{\alpha}{1 + 2\alpha\Omega + \alpha^2}\left[(\Omega + \alpha)(2\ln\alpha - 1) + F(s)\right], \end{aligned} \tag{B.2}$$

where

$$F(s) = \begin{cases} \sqrt{\Omega^2 - 1}\left[\ln\left(-\sqrt{\Omega^2 - 1} - \Omega - \alpha\right) - \ln\left(\sqrt{\Omega^2 - 1} - \Omega - \alpha\right)\right], & \Omega < -1, \\ 2\sqrt{1 - \Omega^2}\arccos\left(-\frac{\Omega + \alpha}{\sqrt{1 + 2\alpha\Omega + \alpha^2}}\right), & -1 < \Omega < 1, \\ \sqrt{\Omega^2 - 1}\left[\ln\left(\sqrt{\Omega^2 - 1} + \Omega + \alpha\right) - \ln\left(\Omega + \alpha - \sqrt{\Omega^2 - 1}\right) - 2i\pi\right], & \Omega > 1. \end{cases} \tag{B.3}$$

In a more compact way, one can rewrite this by keeping the $i\varepsilon$, $\varepsilon \to 0^+$, explicitly, which is necessary to choose the correct Riemann sheet,

$$F(s) = \sqrt{\Omega^2 - 1}\left[\ln\left(-\sqrt{\Omega^2 - 1} - \Omega - \alpha - i\varepsilon\right) - \ln\left(\sqrt{\Omega^2 - 1} - \Omega - \alpha + i\varepsilon\right)\right]. \tag{B.4}$$

$I^{(1)}(s)$ is defined as

$$P^\mu I^{(1)}(s) = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{k^\mu}{(M^2-k^2)\left[m^2-(P-k)^2\right]}. \tag{B.5}$$

One gets

$$I^{(1)}(s) = \frac{1}{2s}\left[(s-m^2+M^2)I(s)+\Delta_\pi-\Delta_{\Sigma_Q}\right], \tag{B.6}$$

where in the IR method [44],

$$\begin{aligned}\Delta_\pi &= i\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-M^2} = 2M^2\left(\bar{\lambda}+\frac{1}{16\pi^2}\ln\alpha\right),\\ \Delta_{\Sigma_Q} &= i\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2} = 0\ .\end{aligned} \tag{B.7}$$

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